

* MATRIX REPRESENTATION OF CONVOLUTIONAL CODE

Generator
matrix



Consider the input bit sequence,

$x(0) \ x(1) \ x(2) \ x(3) \dots$

input Bit Sequence.

row vector

$$x(-1) = x(-2) = 0$$

$$[x(0) \ x(1) \ x(2) \ x(3) \ \dots \dots \dots]$$

$$C_0(0) \ C_1(0) \ C_2(0) \ C_1(1) \ C_2(2) \ C_1(2) \ C_0(3) \ C_1(3)$$

$x(0)$	1	1	0	1	1	1	0	0
$x(1)$	0	0	1	1	0	1	1	1
$x(2)$	0	0	0	0	1	1	0	1
$x(3)$	0	0	0	0	0	0	1	1
$x(4)$	$\vdots$	$\vdots$	0	0	0	0	0	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

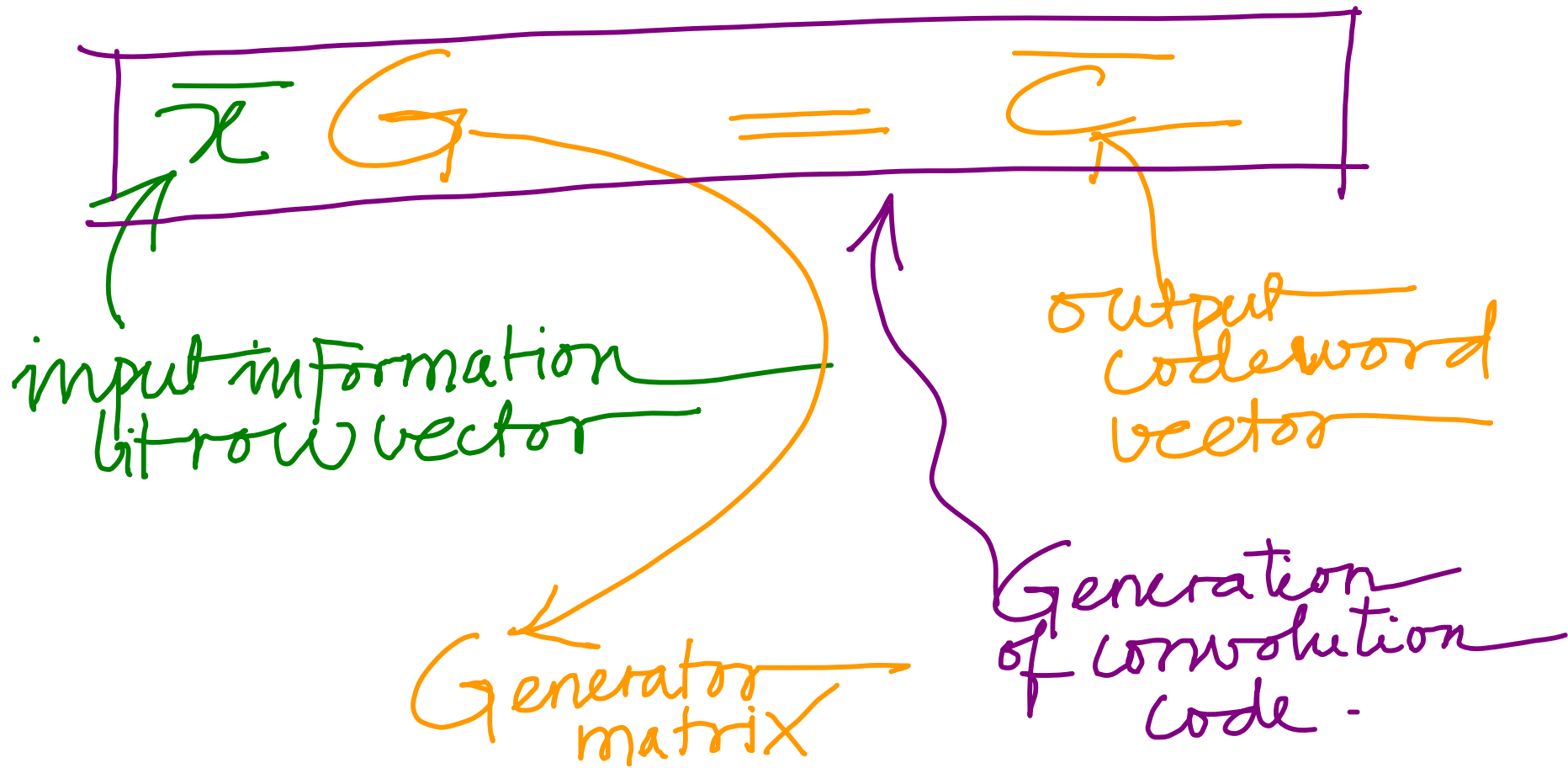
Generator matrix

code bit at time 3

$$C_0(3) = 1 \cdot x(3) + 0 \cdot x(2) + 1 \cdot x(1) = x(3) + x(1)$$

Block shifted downward by 1 in every time instant

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$



TRANSFORM DOMAIN

RERESENTATION:

Define, D-Transform
of bit sequence

$$X(D) = \sum_{j=0}^{\infty} x(j) D^j$$

$\nearrow \sum_{j=0}^{\infty} x(j) Z^{-j}$
Z Transform

↓ Obtained by $D = Z^{-1}$
Delay Element

$$x(j) \longleftrightarrow X(D)$$
$$x(j-k) \longleftrightarrow ?$$

$$\sum_{j=0}^{\infty} x(j-k) D^j$$
$$= \sum_{j=k}^{\infty} x(j-k) D^j$$

since $x(-1), x(-2) \dots = 0$

$$\begin{aligned}
 j-k &= \tilde{j} \\
 \Rightarrow j &= \tilde{j} + k \\
 &= \sum_{\tilde{j}=0}^{\infty} x(\tilde{j}) D^{(\tilde{j}+k)} \\
 &= D^k \cdot \underbrace{\sum_{\tilde{j}=0}^{\infty} x(\tilde{j}) D^{\tilde{j}}}_{X(D)}
 \end{aligned}$$

$$\boxed{x(j-k) \longleftrightarrow D^k X(D)}$$

$$x(j-1) \longleftrightarrow (D) \cdot X(D)$$

✓ D represents a delay of 1 time instant

Convolutional Code:

$$C_0(j) = x(j) + x(j-2)$$

$$C_0(D) = X(D) + D^2 X(D)$$

$$\boxed{C_0(D) = X(D)(1 + D^2)}$$

$$G(j) = x(j) + x(j-1) + x(j-2)$$



$$G(D) = X(D) + DX(D) + D^2 X(D)$$

$$= X(D) \cdot (1 + D + D^2)$$

$$C_0(D) = X(D) \cdot (1 + D^2)$$

$$G(D) = X(D)(1 + D + D^2)$$

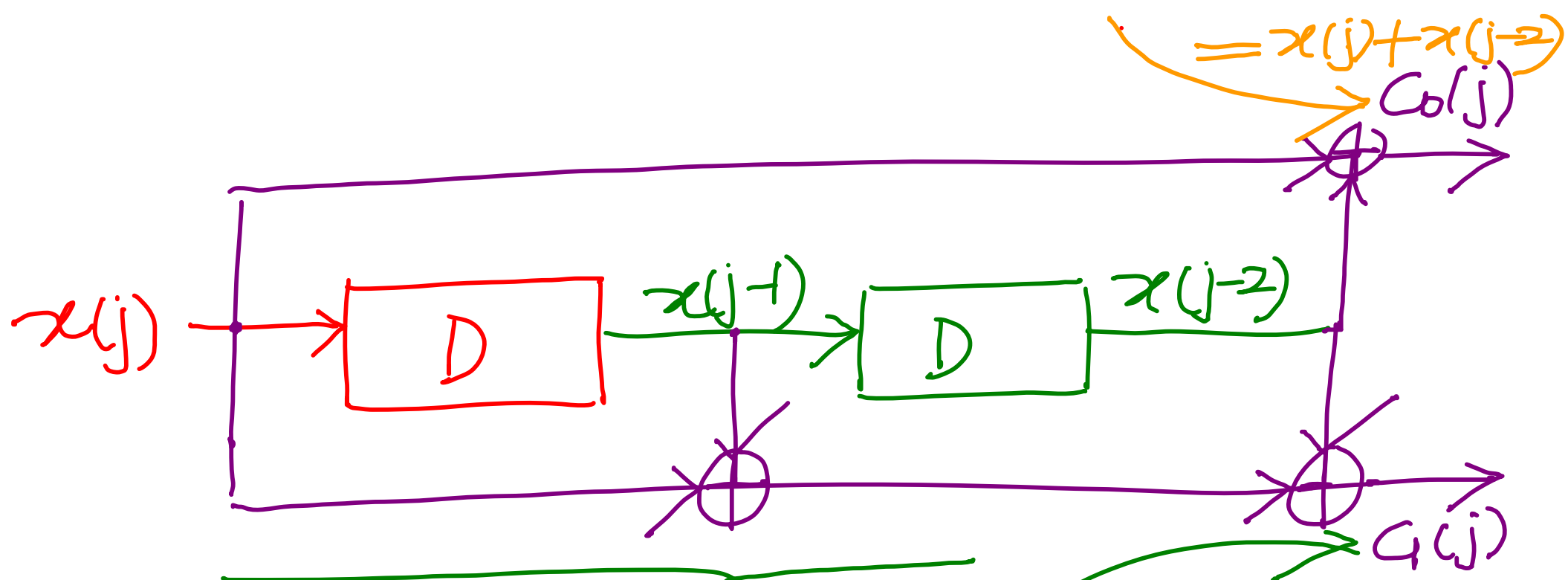
Shift Register

Architecture:

Convolutional code
can be represented using
a shift register architecture.

Feed Forward shift
register architecture is
given below.

code bit c_j at
time t



Feed Forward Shift Register Architecture.

Codebit 1
at time j